**Section 20.120 Mathematics Standards for Elementary Teachers**

a) Core Content Area Knowledge

1) College Algebra

Effective elementary teachers:

A) identify, solve and apply linear and absolute value equations and inequalities;

B) identify and interpret the domain, inverse (if it exists) and graph polynomial, rational, exponential and logarithmic equations;

C) identify the sum, difference, quotient, product of two functions and the resulting domain;

D) identify the composition of two functions and the resulting domain;

E) identify and solve polynomial, rational, exponential and logarithmic equations and inequalities, and apply these methods in solving word problems;

F) solve systems of linear equations using the augmented matrix method;

G) recognize and graph conic sections; and

H) input and interpret data and use technology to find the appropriate regression.

2) Statistics

Effective elementary teachers:

A) construct, identify and interpret frequency distributions, histograms, cumulative frequency tables, ogives and box plots;

B) identify, calculate and interpret measures of central tendency and dispersion;

C) identify, calculate and apply the methods of counting;

D) identify, calculate and interpret probabilities and expected value;

E) define random variables as well as analyze and interpret the probability distributions they generate;

F) identify and describe the sampling distribution of sample means and sample proportions;

G) create and interpret confidence intervals for single population means and proportions;

H) identify, analyze and perform formal tests of hypotheses concerning single population means and single population proportions; and

I) identify, calculate and interpret the correlation coefficient and regression equations.

b) The Mathematics Curriculum

Effective elementary teachers:

1) understand the Illinois Learning Standards for Mathematics (see 23 Ill. Adm. Code 1.Appendix D), their organization, progressions and the interconnections among the domains; and

2) know the developmental sequence of mathematics skills, along with age-level or grade-level benchmarks of development.

c) Foundational Knowledge

1) Standards for Mathematical Practice

Effective elementary teachers enable students to acquire the skills necessary for strong mathematical practice in that they are able to:

A) make sense of problems and persevere in solving them;

B) reason abstractly and quantitatively;

C) construct viable arguments and critique the reasoning of others;

D) model with mathematics;

E) use appropriate tools strategically;

F) attend to precision;

G) look for and make use of structure; and

H) look for and express regularity in repeated reasoning.

2) Counting and Cardinality

Elementary teachers are prepared to develop student proficiency and address common misconceptions related to counting and cardinality and:

A) Demonstrate an understanding of the intricacy of learning to count, assisting students to:

i) know the names of numbers and orally present them in order, starting from the numeral 1 and from various other numbers; being able to recognize written numerals and the quantity each represents; and knowing the names of numbers, starting with eleven, with special attention paid to helping students understand the differences between numbers ending in "teen" and those ending in "ty";

ii) count the number of objects using one-to-one correspondence, regardless of the way in which the object is arranged, and understand cardinality (connecting number name to quantity, the last number of the count, and nesting of numbers) to counting out a given number of objects; and

iii) compare numbers by matching quantity represented with objects or pictures or written numerals; and

B) recognize the role of ten and the difficulties English language learners face because the base-ten structure is not evident in all of the English words for numbers.

3) Operations and Algebraic Thinking

Elementary teachers are prepared to develop student proficiency and address common misconceptions related to operations and algebraic thinking and:

A) solve addition, subtraction, multiplication and division problems with unknowns in any position;

B) demonstrate an understanding of addition and subtraction relationships and multiplication and division relationships, including the use of properties of operations (i.e., the field axioms);

C) demonstrate an understanding of the equal sign as meaning "the same amount as" rather than "calculate the answer";

D) demonstrate an understanding of the meaning of 0 and why division by 0 leads to an undefined answer;

E) understand and apply the meaning and uses of remainders, factors, multiples, parentheses and prime and composite numbers;

F) recognize the following strategies when using the operations of addition and subtraction: counting all, counting on and converting to an easier problem by composing or decomposing ten;

G) recognize extensions of the strategies enumerated in subsection (c)(3)(F) of this Section in multiplication, division and beginning work in expressions and equations;

H) strategically use algebraic tools, such as tape diagrams, number lines, bar models, math racks and double number lines;

I) extend understanding of arithmetic and operations to algebraic expressions and equations, and solve one-step and two-step equations and inequalities; and

J) view numerical and algebraic expressions as "calculation recipes", describing them in words, parsing them into their component parts, and interpreting the components in terms of a context.

4) Numbers and Operations in Base Ten

Elementary teachers are prepared to develop student proficiency and address common misconceptions related to numbers and operations in base ten and:

A) understand how the place value system relies on repeated groupings of any fixed natural number quantity (including ten) and can demonstrate how to use oral counting, objects, drawings, layered place value cards and numerical expressions to help reveal place value structure;

B) understand how to compare numbers, fractions and decimals using the symbols for "greater than", "less than" and "equal to";

C) understand composing and decomposing numbers using the commutative, associative and distributive properties to efficiently use place value methods for addition, subtraction, multiplication and division;

D) extend place value system knowledge to decimals and view decimals as numbers that can be placed on number lines and explain the rationale for decimal computation methods;

E) understand and distinguish between the appropriate use of computation strategies and computation algorithms, as defined in the Illinois Learning Standards for Mathematics, recognizing the importance of "mental math" and derive various algorithms and recognize these as summaries of reasoning, rather than rules;

F) extend place value system knowledge to negative, rational and irrational numbers; and

G) use mathematical drawings, manipulative materials or mathematical properties to reveal, discuss and explain the rationale behind, as well as validate or dismiss, any computational algorithm that a student might present.

5) Number and Operations – Fractions

Elementary teachers are prepared to develop student proficiency and address common misconceptions related to numbers and operations involving fractions and:

A) understand and apply fractions as numbers that can be modeled from a length perspective (number line), an area perspective (pattern blocks, geoboards, etc.), and a discrete perspective (set of dots or circles);

B) understand and apply the concept of unit fractions, benchmark fractions and the whole (referent unit) as defined in the Illinois Learning Standards for Mathematics;

C) extend the associated meanings of the properties of operations from whole numbers to fractions;

D) understand and use equivalent fractions, including those of whole numbers, to reveal new information and as a tool for comparison or to perform operational procedures;

E) understand and apply the connection between fractions and division, and demonstrate how fractions, ratios and rates are connected via unit rates;

F) demonstrate an understanding of decimal notation for fractions, and compare decimal fractions;

G) represent ratios and equivalent ratios as an application of equivalent fractions, and solve ratio and rate problems using tables, tape diagrams, number lines and double number lines;

H) understand the connection between a proportional relationship and a linear relationship, and recognize the connection between an inversely proportional relationship and a reciprocal relationship;

I) defend the ordering of a list of fractions using common denominators, using common numerators, comparing to benchmark fractions or using reasoning; and

J) understand the connection between fractions and decimals, particularly with regard to decimal computations.

6) Measurement and Data

Elementary teachers should be prepared to develop student proficiency and address common misconceptions related to measurement and data and:

A) understand and apply the general principles of measurement; that is, measurement requires a choice of measurable attribute, that measurement is a comparison with a unit and how the size of a unit affects measurements, and the iteration, additivity and invariance used in determining measurement;

B) recognize and demonstrate the relationship of different units;

C) connect the number line to measurement;

D) demonstrate an understanding of area and volume and give rationales for area and volume formulas that can be obtained by compositions and decompositions of unit squares or unit cubes;

E) use data displays to ask and answer questions about data;

F) understand the measures used to summarize data, including the mean, median, interquartile range and mean absolute deviation, and use these measures to compare data sets;

G) examine the distinction between categorical and numerical data and reason about data displays; and

H) recognize the connection of categorical and measurement data to statistical variability and distributions.

7) Geometry

Elementary teachers should be prepared to develop student proficiency and address common misconceptions related to geometry and:

A) compose and decompose shapes and classify shapes into categories, and justify the relationships within and between the categories;

B) understand geometric concepts of angle, parallel and perpendicular, and use them to describe and define shapes;

C) describe and reason about spatial locations (including the coordinate plane);

D) reason about proportional relationships in scaling shapes up and down;

E) describe the connections (relationships) between geometric properties and arithmetic and algebraic properties, and adapt a problem in one domain to be solved in the other domain;

F) summarize and illustrate the progression from visual to descriptive to analytic to abstract characterizations of shapes; and

G) use the coordinate plane to graph shapes and solve problems.

d) Using High-Leverage Instructional Practices

Effective elementary teachers:

1) choose and use mathematical tasks that entail complex mathematical work, build basic skills and allow for multiple answers or methods;

2) teach and use the content-specific language of mathematics;

3) lead whole-class math discussions (e.g., number talks) that engage all learners;

4) respond productively to students' "errors" by probing the underlying thinking and providing targeted feedback;

5) appraise, choose and modify tasks and texts for a specific learning goal;

6) use specific mathematically focused positive reinforcement;

7) use public recording (posters, whiteboard) to collect and probe mathematical thinking (e.g., demonstrating multiple answers and methods; exploring when an algorithm may be the best solution and when another approach may provide an easier solution);

8) diagnose common (and not so common) patterns of student thinking; and

9) assess students' mathematical proficiency and teach responsively.

e) Using Materials, Tools and Technology

Effective elementary teachers:

1) apply mathematical content and pedagogical knowledge to select and use instructional tools, such as manipulatives and physical models, drawings, virtual environments, spreadsheets, presentation tools, websites and mathematics-specific technologies (e.g., graphing tools and interactive geometry software), recognizing both the insight to be gained and any limitations;

2) empower students to make sound decisions about the appropriate use of mathematical tools;

3) when making mathematical models, recognize that technology can enable one to visualize the results of varying assumptions, explore consequences, examine characteristics and compare predictions with data;

4) select mathematical examples that address the interests, backgrounds and learning needs of each student; and

5) evaluate curricular materials for appropriate level and depth of content, focus on and relevance to required learning goals, and incorporation of the Illinois Learning Standards for Mathematics.

f) Monitoring Student Learning through Assessment

Effective elementary teachers:

1) engage in purposeful classroom assessment aligned to appropriate learning expectations for every student and monitor student progress in meeting developmental benchmarks in mathematics;

2) provide a variety of well-designed one-step, two-step and complex multi-step assessment items and performance tasks, incorporating real-life situations to allow students to demonstrate their learning;

3) ensure that assessments are responsive to, and respectful of, cultural and linguistic diversity and exceptionalities, and are not influenced by factors unrelated to the intended purposes of the assessment;

4) guide students in developing the skills and strategies to assess their work and set appropriately ambitious goals for their progress as mathematicians;

5) analyze student work to determine misunderstandings, misconceptions, predispositions and newly developing understandings, and use the results of this analysis to guide instruction and provide meaningful feedback; and

6) communicate the purposes, uses and results of assessments appropriately and accurately to students, parents and colleagues.

g) Meeting the Needs of Diverse Learners

Effective elementary teachers:

1) understand the impact of cultural, linguistic, cognitive, academic, physical, social and emotional differences on mathematics development and progression of knowledge;

2) plan and implement mathematics instruction that capitalizes on strengths and is responsive to the needs of each student;

3) use a variety of approaches and classroom-based intervention strategies to respond to the needs of struggling and/or advanced learners;

4) seek appropriate assistance and support for struggling and/or advanced learners;

5) collaborate and plan with other professionals to deliver a consistent, sequenced and supportive instructional program for each student;

6) differentiate strategies, materials, pace and levels of cognitive complexity to introduce concepts and skills to meet the learning needs of each student; and

7) make content accessible in appropriate ways to English language learners and students with exceptionalities.

h) Constructing a Supportive Mathematics Environment

Effective elementary teachers:

1) create an environment that empowers every student to engage in the practice set forth in subsection (c)(1) of this Section;

2) motivate and engage students by designing learning experiences that build self-direction, perseverance and ownership of mathematics;

3) guide students to work productively and collaboratively with each other to achieve mathematics learning goals by using a strategic combination of individual, group and whole-class instruction to meet the learning needs of each student efficiently and effectively;

4) provide tools that are accessible and developmentally appropriate;

5) establish norms and routines for classroom discourse that allow for the respectful analysis of mistakes and the use of mathematical reasoning for mindful critique and argument; and

6) create opportunities and expectations that all students use appropriate written and oral mathematical language, including English language learners and students with exceptionalities.

i) Professionalism, Communication and Collaboration

Effective elementary teachers:

1) continually engage in intensive, ongoing professional growth opportunities that serve to increase mathematical knowledge for teaching, such as lesson study or continuing coursework;

2) analyze instruction for the purpose of self-reflection and making improvements and make use of strategies such as journal writing, video self-analysis and peer observation;

3) communicate and collaborate with other professionals, such as within a professional learning community, to plan teaching, discuss student needs, secure special services for students and manage school policies; and

4) communicate and collaborate with families to support student needs and discuss student progress.